Game Theory with Imprecise Probabilities

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Overview

- We propose two game-theoretic solution concepts based on two preliminary investigations into the issue of introducing imprecise probabilities into games:
    1. **Γ-maximin rationalizability**: Just like rationalizability assuming that all the players are commonly known as expected utility maximizers, it captures the idea that each player considers the other players as Γ-maximin decision makers under uncertainty. (Poster Session on Monday)
    2. **Robustness** under linear tracing procedure (LTP): to modify LTP (Harsanyi and Selten, 1988) by using a set of probability distributions to represent players' initial beliefs about her opponents' strategy choices.

Linear Tracing Procedure

- LTP can be regarded as a rational deliberation process which selects a less risky equilibrium as outcome.
- Starting with a common prior distribution, which represents their initial uncertainty, all players gradually change their own tentative strategy plans, as well as their expectations about the other players' possible strategies, until they arrive at a certain Nash equilibrium.

	s_{21}	s_{22}
s_{11}	$1,1$	$0,0$
s_{12}	$0,0$	$3,3$

Table: Coordination Game

Nash equilibria: $A = (s_{11}, s_{21})$, $C = (s_{12}, s_{22})$, and $E = ((\frac{3}{4}, \frac{1}{4}), (\frac{3}{4}, \frac{1}{4}))$.

An Example of Linear Tracing Procedure

- LTP proposes to examine a family of auxiliary games closely related to the game, which are solved by considering Nash equilibrium as well.
- Note that there is only one feasible path (the blue line) continuously connecting all the auxiliary games, and thus the equilibrium $C = (s_{12}, s_{22})$ is selected as the outcome.

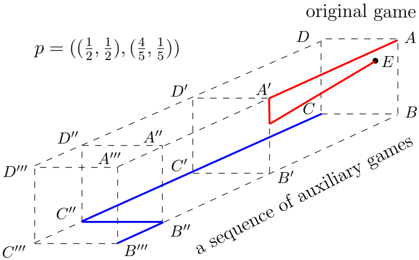

Figure: LTP starting with p

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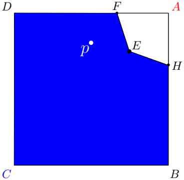

Prior Strategy Space

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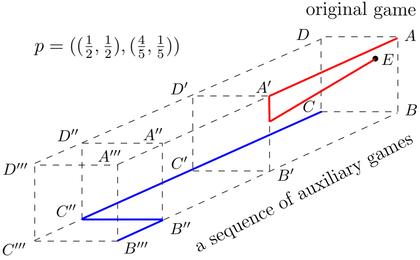

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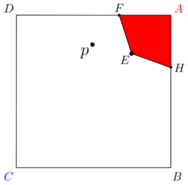

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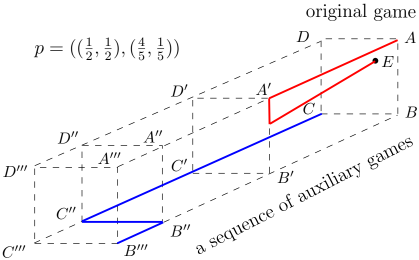

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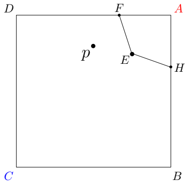
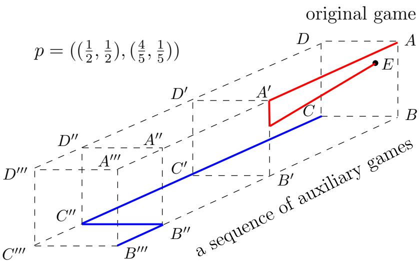


Prior Strategy Space

LTIP with IP

- We revise LTP by using a (common) **set of prior distributions** to describe the players' initial beliefs about other players' strategy choices.
- Suppose that the players' initial belief is represented by a set of prior strategies $\mathcal{P}'$ as shown below (the red area).



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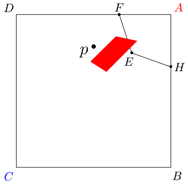
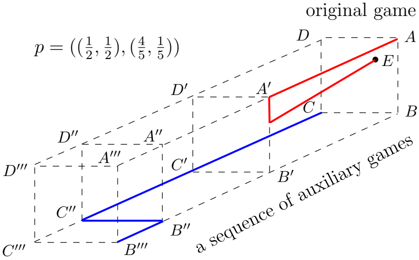


Stability and Robustness

- The notion of **stability** captures the idea of the duration in which a prior leads to the same equilibrium when LTP is iteratively applied to those auxiliary games.
- Then we define a concept of **robustness** by using the maximin decision rule, which can be regarded as a measure for assessing the equilibria.



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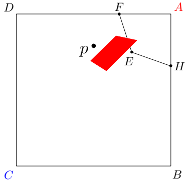
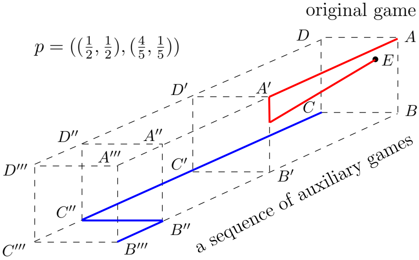


Example

- The most robust equilibrium is the one that maximizes the possible minimum stability of the prior strategies that lead to that equilibrium as outcome.
- In this case the equilibrium A has the largest robustness index, and thus distinguishes itself from the other two equilibria w.r.t. $\mathcal{P}'$.



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